Comment on “Detecting the Kondo Screening Cloud around a Quantum Dot”

A. A. Zvyagin[1]

[1]B. I. Verkin Institute for Low Temperature Physics and Engineering of the NAS of Ukraine, Kharkov, 61164, Ukraine

(Dated: February 1, 2008)

We point out several mistakes in the recent work of I. Affleck and P. Simon (cond-mat/0012002).

PACS numbers: 72.10.Fk, 72.15.Qm, 73.23.Ra

Recently [1] it was claimed that the effect of the Kondo screening on persistent currents (PC) was studied in a mesoscopic ring coupled to a quantum dot. The system was considered in the framework of the Anderson impurity (AI) model for two situations — for large Kondo length scales $\xi_K \gg L$ and small ones $\xi_K \ll L$. Here $\xi_K = \hbar v_F/T_K$, v_F is the Fermi velocity of electrons in the ring and T_K is the Kondo energy scale. We want to point out the following:

(i) Charge degrees of freedom (and, hence, PC) are *totally decoupled* from spin degrees of freedom in the “bare” Kondo problem [2]. Hence, the Kondo screening itself *does not* affect (except of initial phase shifts) charge PC (the Aharonov-Bohm effect), but rather spin PC (the Aharonov-Casher one) [3]. On the other hand, the AI (for which both spin *and charge* degrees of freedom are hybridized with the host) really influences charge PC [4].

(ii) In the case $\xi_K \gg L$ (i.e., T_K being much less than energy spacings of electrons in the ring $\hbar v_F/L$) one can neglect all other levels of electrons in the ring except of the lowest. (Sinusoidal oscillations of PC with the magnetic flux obviously results.) In this case the renormalization of the ground state energy of the localized electron due to the hybridization with ones from the ring (i.e., T_K [2]) is *not* determined by Eq. (1) of [1]. In the limit $|\epsilon_0|, U \gg t'$ [1] it is rather proportional to $2t'^2/|(\hbar v_F/L) + [(U+\epsilon_0)\epsilon_0/U]|$, where t' are hopping elements of dot-ring contacts, U and ϵ_0 are the standard parameters of the AI Hamiltonian [1, 2]. Kondo logarithms (which appear if one takes into account *thermodynamically large number* of states in the Fermi sea of ring electrons) are absent in this case [2]. Here one can speak about the Kondo effect only in a “Pickwick sense” (the Abrikosov-Suhl resonance is too far from the Fermi energy and too wide). The other situation $\xi_K \ll L$ ($T_K \gg \hbar v_F/L$) was studied *exactly* 6 years ago in [3, 4] (unlike the *qualitative* study of this case in [1]). Here a thermodynamically large number of states of electrons of the ring produces exponential, generic for the *correlated nature* of the Kondo effect, dependence for T_K [2]. Hence, in the ground state the oscillations of PC are “saw-tooth”-like (the sum of many harmonics) [3, 4, 5]. Their magnitude is proportional to $ev_F/\hbar L$ [4, 5]. However the conclusion of [1] that PC in this case are those of *ideal* ring (i.e., without dot) is invalid. The intra-dot Coulomb repulsion manifests itself in the matrix of “dressed charges” (its components measure numbers of electrons which form low-lying charge and spin excitations of a system), being *different* from unity matrix (noninteracting case) [3, 4]. Only in the limit of *zero* magnetization the behavior of PC is reminiscent of the one for free electrons, while for any nonzero Zeeman splitting (which is the generic situation) it differs drastically. Moreover, the single velocity (v_F) present in the answer for PC is the consequence of the linearization of the dispersion law for electrons in the ring in the AI model. It is easy to show that the study of PC in a *lattice* Bethe ansatz-solvable model (e.g., [6] where the AI-like impurity was situated in the correlated electron ring) produces the answer for the PC similar to the ones for the AI model [4], but with *two different velocities* for low-lying spin and charge excitations. The latters become equal to v_F only if one linearizes the spectrum of the host.

(iii) PC for the *multichannel* Kondo impurity were *exactly* studied in [7] (notice that the results of [7] can be used for chiral fermions, too). Here multichannel Kondo screening also *does not* affect charge PC (as the consequence of the spin-charge separation) for *any* values of the local exchanges between the localized spin and channel electrons of the ring. However the correlation effects induced by the magnetic impurity introduce the interference of several Aharonov-Casher oscillation patterns for spin PC. There is also an interference between the Coulomb blockade-like (parity) effects and PC oscillations [7].

This has been the text of my manuscript submitted to the Physical Review Letters on March 26, 2001. Several remarks are in order to clarify the brief above argumentation.

1. A number of recent publications considered persistent currents in quantum rings with magnetic impurities. There is a principal difference between persistent currents and usual transport currents. Recall, usual currents are *transport*, kinetic characteristics of any system. Their usual characteristics are the resistivity or conductivity and related to them transition amplitude. In the framework of the linear response theory the conductivity is the coefficient, which connects the value of the current with the value of an applied *electric* field. Hence, one can consider transport currents in a system only if it has the sourse and drain, cf. Fig. 1 (a), and the transport current is the consequence of the difference in potentials applied

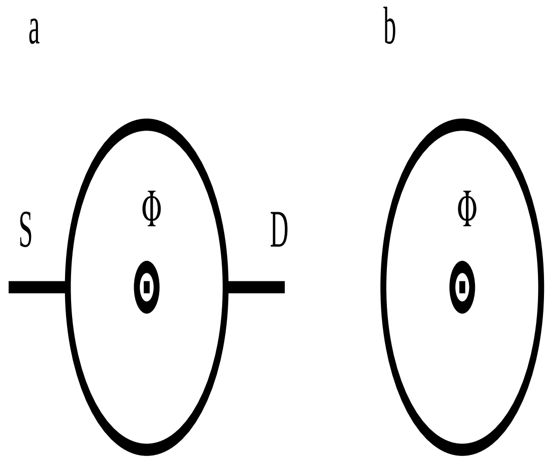

FIG. 1: Different geometries for the manifestation of the Aharonov-Bohm effect of an external magnetic flux Φ in a metallic ring: (a) the transport current geometry with the source (S) and drain (D); (b) the persistent current geometry.

to the sourse and drain. One necessary needs this difference to study any transport current. On the other hand, the persistent current is the *thermodynamic* characteristic of a ring. It is connected with the Aharonov-Bohm phase shift, which appears when charges move along a loop, pierced by a magnetic flux [8]. Then an external magnetic flux yields nonzero momentum of charges. In fact the persistent current is nothing else than the *total orbital moment* of all charges in the ring. Naturally, the persistent current can exist without any applied external electric field; it does not need any sourse and drain, cf. Fig. 1 (b). Persistent current is just the derivative of the energy of a system in equilibrium with respect to the applied magnetic flux. For noninteracting electron rings the charge persistent current is connected with the virtual movement of an electron around the magnetic flux. For interacting electron systems charge persistent currents pertain to the virtual movement(s) of charge-carrying excitations (in the ground state and at low temperatures — to the virtual movement of the low-lying charged excitations). Clearly, in the situation of Fig. 1 (b) (without any sourse and drain) it is inappropriate to speak about the conductance/conductivity or resistivity of a system — at least one should first properly define, what are the latters. The missunderstanding sometimes appears because several recent experiments on quantum rings studied namely the geometry of Fig. 1 (a), but not of Fig. 1 (b) (i.e., they measured transport currents between sourses and drains, but not the total orbital moment of the ring). Naturally, when the ring between the sourse and drain is pierced by an external magnetic flux in the geometry of Fig. 1 (a), the transport current is also affected by that flux. Hence, the conductance of the transport current also becomes flux-dependent. However this transport current is naturally *not* exactly equal to the persistent current. It turns out that the authors of Ref. [1] definitly wrote about persistent currents, not transport currents (cf., Fig. 1 and Fig. 1 of [1]).

This difference in the basic features of transport and persistent currents produces the main difference in the answers, when one considers the effect of a magnetic (Kondo) impurity. The Kondo impurity introduces the interaction into the free electron problem [2]. It is well known that in the pure Kondo problem (a magnetic impurity, just a spin, coupled to the free electron host via a local exchange interaction) collective spin and charge degrees of freedom decouple from each other [2]. Notice that in the Anderson impurity model the hybridization impurity manifests both charge and spin degrees of freedom [2]. On the one hand, the resistivity of a transport current is strongly affected by the Kondo impurity [2], because the resistivity is determined by the density of *all* states of the system, spin and charge. The Kondo (Abrikosov-Suhl) resonance (which is the characteristic feature of *spin* degrees of freedom, cf., [2, 9]) determines the magnetoresistivity of the transport current. Hence, the conductance of a quantum ring [in the geometry of Fig. 1 (a)] with embedded magnetic (Kondo) impurity is, obviously, affected by that impurity. On the other hand, the persistent current [i.e., the total orbital moment of a quantum ring in the geometry of Fig. 1 (b)] is determined by the virtual movement of only *charge-carrying excitations* around the magnetic flux. In the pure Kondo situation, in which a magnetic impurity (spin) is connected to the free electron host via a local spin exchange, the charge degrees of freedom are mostly *not* affected by the spin impurity [2] (the only possible effect is the possible presence of an initial phase shift, see 4. below). Hence, in this case the Kondo impurity does not influence the frequency and the magnitude of persistent currents in the geometry of Fig. 1 (b). Namely this property of metallic rings with embedded spin (Kondo) impurity was pointed out in our pioneering Letter of 1994 [3], which studied the effect of the Kondo impurity on persistent currents for the first time. The situation is very diffrent for the Anderson (hybridization) impurity. Here the impurity affects both spin and charge low-lying excitations [2]. In this case the Anderson impurity affects charge persistent currents of the geometry of Fig. 1 (b) [4]. For example, the Anderson impurity produces low-lying excitations, which carry charge $-2e$ (spin-singlet bound states of electrons) and excitations, which carry spin 1/2 and charge $-e$ [2]. The virtual movement of excitations, which carry charge $-2e$, naturally yields oscillations of charge persistent currents with the period $\Phi_0/2$ (where $\Phi_0 = hc/e$), while unbound electron excitations, which carry charge $-e$, produce oscillations of persistent currents with the period Φ_0 [4]. For the case with the dispersion law of host electrons being linearized about Fermi points the velocities of both types of low-lying excitations are equal

to each other (and both are equal to the Fermi velocity) [2]. In the absence of the Zeeman effect of the external magnetic field the interference of those two type of oscillations of persistent currents produces oscillations with the period Φ_0, reminiscent of the ones in a free electron host. (Notice also the parity effect of persistent currents in a ring with the magnetic impuritry, i.e., different initial phases [dia- or paramagnetic persistent currents] and periodicities for different numbers of electrons in the ring, predicted in [3, 4, 5, 7]). However the nonzero curvature of the spectrum of host electrons and the Zeeman effect of the applied magnetic field (it is small, naturally, for the case of GaAs-based quantum rings, where effective g-factors are small) must produce the real interfererence of two types of oscillations of charge persistent currents (with the periods Φ_0 and $\Phi_0/2$), because their magnitudes become different from each other in that case. It turns out that we pointed out the difference between the characteristics of persistent currents and transport currents of a metallic ring with a Kondo impurity in Ref. [7], where it is clearly shown that a magnetic Kondo impurity (spin) does not change the properties of charge persistent currents, but drastically affects the magnetoresistivity for charge transport currents, cf. subsections A and B of the Section IV of [7].

2. To explain other inconsistencies of [1] let us start with the description of the low-energy behavior of the Anderson model. In this explanation we use different from [2], variational approach to the Kondo problem [10]. This approach is well-known and later was reproduced in a number of books and textbooks, see e.g., [11, 12]. Naturally similar results can be obtained using the Bethe ansatz approach to the Kondo problem [2], or using other approaches, like the non-crossing-like approximations [13] or the slave-boson technique [14]. For simplicity we consider the Anderson impurity Hamiltonian with $U \to \infty$ (similar results can be obtained for finite U). The variational wavefunction for the ground state can be taken as

$$|\psi\rangle = A(|0\rangle + \sum_{\epsilon} a(\epsilon)|\epsilon\rangle) \ , \tag{1}$$

where $|0\rangle$ determines the state of host electrons, in which all states below the Fermi energy are occupied and the impurity level is empty, $|\epsilon\rangle$ is the state with one electron at the impurity level and one hole below the Fermi energy, $a(\epsilon)$ is the variational coefficient, and A is the normalization constant. The minimization of the ground state energy with respect to a yields the equation

$$\Delta E = t'^2 \sum_{\epsilon} \frac{2}{(\Delta E - \epsilon_0 + \epsilon)} \ , \tag{2}$$

where we introduced ΔE, the renormalization of the ground state energy due to the Anderson impurity. If the number of states in the sum is thermodynamically large (namely it is the case for the small ratio $\hbar v_F/L$), one can replace the sum by the integral, which for the constant density of states of conduction electrons $N(0)$ yields the famous Kondo logarithm and one obtains:

$$\Delta E = -De^{-\frac{|\epsilon_0|}{2N(0)t'^2}} = -De^{-\frac{1}{N(0)J_{eff}}} \equiv -T_K \tag{3}$$

(D pertains to the bandwidth of conduction electrons). It means that T_K is namely the renormalization of the ground state energy due to the hybridization Anderson impurity. This, naturally, agrees with other definitions of the Kondo scale, see, e.g., Refs. [2]. This equation is equivalent, naturally, to Eq. (1) of [1]. In this case, obviously, the thermodynamically large number of harmonics produces the "saw-tooth"-like oscillations of persistent currents, which were obtained in Refs. [3, 4, 5, 7] and (partially) reproduced in [1]. However, it turns out that if the number of states in the sum is small (which is the case for the large ratio $\hbar v_F/L$), one *cannot* replace the sum by the integral, and the Kondo logarithm does not appear in Eq. (2). One can, naturally, find the renormalization of the ground state energy (which also plays the role of the Kondo temperature in this case), however it will be *not* exponentially dependent on the effective exchange constant J_{eff}. For one electron level of the ring being involved it is namely the answer written above in the text of the Comment (in the limit $U \to \infty$; for finite U one can follow for the variational approach [15]). This is actually the answer obtained in [16], to which the limiting case of the large ratio $\hbar v_F/L$ of [1] pertains ([1] actually introduces only the $O(J^2)$ corrections to the result [16]). Naturally, for only one level of conduction electrons being involved one clearly obtains only one harmonics (i.e., sinusoidal oscillations of the persistent current, cf. [1, 16]). This means that in the limiting case of large $\hbar v_F/L$ one *cannot* use Eq. (1) of [1] for the determination of T_K. Actually this limit ($\xi_K \gg L$) has no other, standard for the Kondo situation, features. For example, it is well known that the generic Kondo effect affects the Sommerfeld coefficient of the low temperature specific heat and magnetic susceptibility of the magnetic impurity: They become large (inverse proportional to the small Kondo temperature). However for the finite number of states of the case $\xi_K \gg L$ the specific heat is exponentially small at low temperatures, and the magnetic susceptibility is determined by the (possible for systems with orbital degrees of freedom) van Vleck (zero temperature) terms, while temperature corrections are exponentially small, too [17].

In Ref. [4] persistent currents were calculated for the ring with the Anderson impurity (which produces effective interactions for *both* spin and charge degrees of freedom). In this paper it is shown that persistent currents are determined by $1/L^2$ corrections to the energy (L is the size of the ring), while the behavior of the impurity itself (its valence, magnetization, etc.) are determined by

$1/L$ corrections, if the main contribution to the energy is of order of 1 (i.e., per site). [This statement is true for *any* one-dimensional quantum ring with gapless low-lying excitations.] The Kondo scale in the magnetic field behavior appears for the characteristics of the impurity, but not (in the main order in $1/L$) for persistent currents (it appears in higher-order corrections, too, but they are irrelevant for our discussion). The main quantities, which determine the values of persistent currents in ideal ballistic quantum mesoscopic rings are the (Fermi) velocities of low-lying excitations, which virtual movement defines the Aharonov-Bohm effect in interacting systems, and the matrix of "dressed charges" (this quantity measures the effective number of "initial" electrons which form each low-lying excitation). In principle both of those quantities depend on the magnetic field (in the Bethe ansatz scheme that dependence is obtained via solutions of integral equations for mentioned quantities). For the model of the Anderson impurity studied in Ref. [4] (with the linearized dispersion law of host electrons) the velocities of low-lying charge and spin excitations coincided with the initial Fermi velocity of electrons and do not depend on the external magnetic field (cf. [2]). Notice that for hybridization impurity models on the lattice, cf. [6], it is not true — there are *two* different from each other velocities of low-lying excitations and both of them depend on the magnetic field. However the matrix of "dressed charges" *does* depend on the magnetic field even for the linearized dispersion law of host electrons. This dependence does not reveal the Kondo scale (in the main order in $1/L$). For *exactly* zero magnetic field the answer for the charge persistent current is reminiscent of the one for a ring of noninteracting electrons (as it must be). However even a small deviation of the value of the field from zero produces very strong changes in the values of the components of the "dressed charge" matrix [because of logarithmic corrections, present in the SU(2)-symmetric system; these corrections are well-known to one of the authors of [1], who published many papers devoted to similar logarithmic corrections [18]]. Hence the persistent current becomes to be different from the one of the ring with free electrons for small $\hbar v_F/L$ (even if the value of the magnetic field is small compared to T_K, and the magnetization of the impurity is small). These corrections, being very small, are not very important for the magnetization of the impurity, however they are of great importance for persistent currents — e.g., the magnitudes of the oscillations of persistent currents with periods Φ_0 and $\Phi_0/2$ will be different even for very small fields, i.e., an additional period of persistent currents can be observed. We emphasize that it is not rare that the effect of an (even small) magnetic field on the quantum dot embedded into the quantum ring is important (cf. [19]).

By the way, in a recent preprint [20] (which appeared after the authors of [1] became aware of my above written text of the Comment) one of the authors of [1] directly wrote (cf. P.7 of [20]): "On the other hand, when $\xi_K \gg L$ the Kondo effect doesn't take place: the infrared divergence of the Kondo coupling, λ, is cut off by the finite size of the ring." This is in obvious contradiction to the statements of [1], where the authors proposed to study the influence of the Kondo effect on persistent currents in *both* cases — $\xi_K \gg L$ and $\xi_K \ll L$ — and obviously coincide with the above mentioned statement of our Comment.

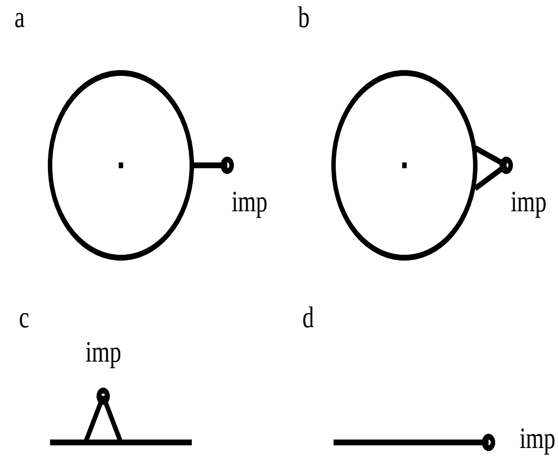

FIG. 2: (a) A side-coupled impurity; (b) an integrable impurity in a ring; (c) an integrable impurity in the bulk of an an open chain; (d) an integrable impurity at the edge of an open chain.

3. The authors of Ref. [1] (and also some others, see, e.g., [21]) claimed that the Bethe ansatz approaches of Refs. [3, 4, 5, 7, 22] considered *side-coupled* quantum dots, see Fig. 2 (a). These claims are absolutely wrong. The Bethe ansatz method can be used either for systems with periodic boundary conditions, or for systems with open boundary conditions (does not matter, whether one studies homogeneous systems or systems with impurities). The Bethe ansatz method is properly justified for discrete coordinates of particles [23]. Field theoretical models, solved by the Bethe ansatz, have to be continuous limits of their lattice counterparts (including the solution of the Kondo problem [2]). (Notice that the continuous limit can be taken in different ways, which introduces some ambiguity.) Impurities can be included into the lattice Bethe ansatz scheme either as shown in Fig. 2 (b) and (c) for the impurity in the bulk of a ring or an open chain, i.e., the impurity is connected with *two* neighboring sites of the host, or connected with only one neigboring site only at the edge of an open chain, as shown in Fig. 2 (d). This is the direct consequence of the fact that impurities can be introduced into the Bethe ansatz monodromies either as special *scattering* matrices (this way implies no reflection in the problem at all) or as local *boundary fields/potentials* (reflectors), which can

be applied *only* to the edges of an open chain. Please, pay attention that we distinguish the reflection and backward scattering (by the latter we mean the transfer from the one Fermi point to another — such processes are present in any lattice integrable theories, like the Heisenberg spin-$\frac{1}{2}$ chain or Hubbard chain [23], but there is no reflection there for periodic boundary conditions). Scattering matrices of impurities have to satisfy *Yang-Baxter* ("triangular", "star-triangle") *relations* [23] with scattering matrices of the host, to preserve the integrability of a problem in the framework of the Bethe's ansatz. Hense, such impurities in periodic Bethe ansatz solvable systems have to be pure scatterers, but must *not* produce any reflection. On the other hand, local fields (reflectrors) can be used only for *open* chains in the lattice Bethe ansatz approach. They are described by reflection matrices, which satisfy *reflection equations* [24]. However for any system with open boundary conditions persistent currents are obviously zero. Hence, the only possibility to study persistent currents in Bethe ansatz-solvable models with impurities is to consider impurities, which produce only scattering phases, but not reflections, and which scattering matrices satisfy Yang-Baxter relations with scattering matrices of the host (and, naturally, mutually). In our papers [3, 4, 5, 7] (see also [25]: It turns out that we studied the problem of the influence of magnetic impurities on charge and spin persistent currents in detail in a large number of refereed papers since 1994 and reported our results at international conferences), devoted to the influence of magnetic and hybridization impurities on persistent currents, we considered only integrable impurities of this class. The Bethe ansatz solution of the Kondo problem [2] also belongs to this class — impurities produces only scattering phases but not reflections. The side-coupled impurity [cf. Fig. 2 (a)] has, naturally, the properties of a reflector (which is correctly pointed out in [1]) and, therefore, cannot be introduced into the Bethe ansatz solvable ring in principle, because it violates the Yang-Baxter relations. This is why, the claim that the side-coupled impurity could be introduced into the Bethe ansatz solvable ring is absolutely incorrect.

4. In the framework of the Bethe ansatz there are two independent very well known solutions to the Kondo problem (pure spin impurity in the free electron host, coupled to the host via the local exchange): the one by N. Andrei and the one by P. Wiegmann [2]. These solutions, being different in some details, produce the same correct answers for thermodynamic characteristics of the Kondo magnetic impurity. Those details are not important for the behavior of the thermodynamic characteristics of the host and the impurity. However, those details can produce the difference in the behavior of finite-size corrections (which namely determine persistent currents in metallic systems). One of those details is the phase factor for the Bethe ansatz equations, which govern the behavior of charge degrees of freedom in the solution of P. Wiegmann (cf., [26]) and the absence of that phase in the approach of N. Andrei (cf., [27]), see also [2]. The presence or absence of those phase factors are determined by the particular choosen schemes of taking the sclaling limit in those two approahes. The Bethe ansatz is developed for systems with *discrete* particles, because of its main property: Any multi-particle scattering process is considered as consequence of pair scattering processes between particles in the Bethe ansatz scheme [23]. Then, when studying continuous limit, one has to use some scaling approximation from the lattice counterpart. Therefore the consideration of the continuous scaling limit for the solution of the Kondo problem has some freedom in the determination of the phase shift. In [3] we used the scaling scheme intoduced by P. Wiegmann. This scheme determined the appearence of the initial phase shift for charge persistent currents, caused by the Kondo impurity. On the other hand, in Ref. [7] we studied persistent currents for a system with multichannel Kondo impurities, and used the scaling scheme introduced by N. Andrei. This is why there was no initial phase shift for charge persistent currents (even for the limiting case of the number of channels being equal to 1). However, as we pointed out above, the presence or absence of that initial phase shift is determined by the (non-controlable) scaling approximation. We do not know, which answer (with or without the phase shift) is generic in the real situation of a quantum dot in a ring. The only argument, which can be used, is that for the *lattice* exactly solvable problem of a magnetic impurity in a correlated electron ring (cf. [6]) the Bethe ansatz equations for charge degrees of freedom *have* phase factors, which, naturally, produce initial phase shifts for charge persistent currents. These initial phase shifts are the consequences of the fact, that a magnetic impurity introduces the nonzero net chirality into the periodic lattice integrable problem.

5. It turns out that our previous results [3, 7] for spin and charge persistent currents in a metallic ring with a magnetic (spin) Kondo impurity coincide with the ones of Ref. [22] and with the limiting case of $\xi_K \ll L$ of [1] for zero magnetization of the system (up to the initial phase shift in [3], see the discusssion in 4. above). In some of our studies we considered the behavior of chiral (only right- or left-moving) electrons. Refs. [1, 22] consider both right- and left-movers together. However the fact that the answers of Refs. [3, 7] and [1, 22] are similar implies that the chirality of host electrons does not play an important role in this problem. By the way, naturally, for the linearized dispersion law of host electrons one must consider for persistent currents only *finite-size corrections*, which depend on external fluxes. The constant term (of order of the size of the system) is the obvious artifact of the linearization of the spectra of host electrons and was, obviously, discarded in Refs. [3, 4, 5, 7, 25].

[1] I. Affleck and P. Simon, Phys. Rev. Lett. **86**, 2854 (2001); cond-mat/0012002.

[2] N. Andrei, K. Furuya and J. H. Lowenstein, Rev. Mod. Phys. **55**, 331 (1983); A. M. Tsvelick and P. B. Wiegmann, Adv. Phys. **32**, 453 (1983); A. C. Hewson, *The Kondo problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993).

[3] A. A. Zvyagin and T. V. Bandos, Low Temp. Phys. **20**, 222 (1994) (notice the typo: it must be $\xi_{SS} = 1/\sqrt{2}$ there).

[4] A. A. Zvyagin, Low Temp. Phys. **21**, 349 (1995).

[5] These results were applied to the ring with a quantum dot in: A. A. Zvyagin, T. V. Bandos and P. Schlottmann, Czech. J. Phys. **46**, Supl. S5, 2409 (1996).

[6] A. A. Zvyagin, Phys. Rev. Lett. **79**, 4641 (1997).

[7] A. A. Zvyagin and P. Schlottmann, Phys. Rev. B **54**, 15191 (1996).

[8] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959); Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).

[9] N. Kawakami and A. Okiji, Phys. Rev. B **42**, 2383 (1990).

[10] C. M. Varma and Y. Yafet, Phys. Rev. B **13**, 2950 (1976); O. Gunnarsson and K. Schönhammer, Phys. Rev. B **28**, 4315, (1983).

[11] P. Fulde, *Electron Correlations in Molecules and Solids* (Springer-Verlag, Berlin, 1993).

[12] G. D. Mahan, *Many-Particle Physics* (Plenum Press, New York, 1990).

[13] N. E. Bickers, D. L. Cox and J. W. Wilkins, Phys. Rev. B **36**, 2036 (1987).

[14] P. Coleman, Phys. Rev. B **29**, 3035 (1984).

[15] O. Gunnarsson and K. Schönhammer, Phys. Rev. B **31**, 4815 (1985).

[16] M. Büttiker and C. A. Stafford, Phys. Rev. Lett. **76**, 495 (1996)

[17] P. Schlottmann, Philos. Mag. Lett. **81**, 575 (2001).

[18] See, e.g., I. Affleck, D. Gepner, H. J. Schultz and T. Ziman, J. Phys. A **22**, 511 (1989); S. Eggert, I. Affleck and M. Takahashi, Phys. Rev. Lett. **73**, 332 (1994).

[19] M. Pustilnik, Y. Avishai and K. Kikoin, Phys. Rev. Lett. **84**, 1756 (2000); T. Costi, *ibid.* **85**, 1504 (2000); J. E Moore and X. -G. Wen, *ibid.* **85**, 1722 (2000); M. Pustilnik and L. I. Glazman, *ibid.* **85**, 2993 (2000).

[20] I. Affleck, cond-mat/0111321.

[21] K. Kang and S. Y. Cho, Phys. Rev. Lett. **87**, 179705 (2001); M. E. Torio, K. Hallberg, A. H. Ceccato and C. R. Proetto, Phys. Rev. B **65**, 085302 (2002).

[22] H.-P. Eckle, H. Johannesson and C. A. Stafford, Phys. Rev. Lett. **87**, 016602 (2001)

[23] See, e.g., V. E. Korepin, N. M. Bogoliubov and A. G. Izergin *Quantum Inverse Scattering Method and Correlation Functions* (Cambridge University Press, Cambridge, 1993) and references therein.

[24] See, e.g., E. K. Sklyanin, J. Phys. A **21**, 2375 (1988).

[25] A. A. Zvyagin and I. N. Karnaukhov, Mod. Phys. Lett. B **8**, 937 (1994); A. A. Zvyagin and T. V. Bandos, JETP Lett. **61**, 682 (1995); A. A. Zvyagin and T. V. Bandos, Mod. Phys. Lett. B **9**, 1253 (1995); A. A. Zvyagin and P. Schlottmann, Phys. Rev. B **52**, 6569 (1995); P. Schlottmann and A. A. Zvyagin, Physica B **230-232**, 624 (1997); A. A. Zvyagin, Mod. Phys. Lett. B **12**, 215 (1998); A. A. Zvyagin, Phys. Rev. Lett. **87**, 179704 (2001).

[26] P. B. Wiegmann, J. Phys. C **14**, 1463 (1981); V. A. Fateev and P. B. Wiegmann, Phys. Lett. **A 81**, 179 (1981); V. M. Filyov, A. M. Tsvelik and P. B. Wiegmann, Phys. Lett. **A 81**, 175 (1981).

[27] N. Andrei, Phys. Rev. Lett. **45**, 379 (1980); N. Andrei and J. H. Lowenstein, Phys. Rev. Lett. **46**, 356 (1981).